0. Definitions

$x \in A$ is true if and only if x is an element of the set A.
$x \notin A$ is true if and only if x is not an element of the set A.
$B \subseteq A$ is true if and only if every element of the set B is an element of the set A.[1]

$\varnothing =_{def} \{ \}$

$A \cap B =_{def} \{ x \mid x \in A \text{ and } x \in B \}$
$A \cup B =_{def} \{ x \mid x \in A \text{ or } x \in B \}$
$A - B =_{def} \{ x \mid x \in A \text{ and } x \notin B \}$
$C(A) =_{def} \{ x \mid x \notin A \}$, we assume that we only consider things in our "discourse universe", U.

$\wp(A) =_{def} \{ B \mid B \subseteq A \}$

1. Different ways of specifying sets

Give a different notation that picks out the same set.

(a) $\{ 2, 4, 6, 8 \}$
(b) $\{ x \mid x^2 = 9 \}$
(c) { Ginger, Greg, Gillian }

For the following exercises, use these sets:

$A = \{$ Q, Patrick, Elisabeth, Jack $\}$
$B = \{ 2, 4, 6, 8 \}$
$C = \{$ {Patrick}, 2, {the Berlin Wall, the Hampshire Mall} $\}$
$D = \{$ Q, 2, 4, Jack, {Elisabeth} $\}$
$E = \varnothing$
$F = \{$ Patrick $\}$

2. Members and subsets

Are the following statements true, false, or ill-formed?

(a) Patrick $\in$ F
(b) Patrick $\in$ C
(c) {Patrick} $\in$ F
(d) {Patrick} $\in$ C
(e) $D \subseteq A$
(f) $B \subseteq D$
(g) $F \subseteq C$
(h) $F \subseteq A$
(i) the Berlin Wall $\subseteq$ C
(j) {the Berlin Wall} $\subseteq$ C
(k) the Berlin Wall $\in$ C
(l) {the Berlin Wall} $\in$ C

[1] This relation is sometimes called 'subset or equal' in order to distinguish it from 'proper subset', written $\subset$. $B \subset A$ is true if and only if every element of B is an element of A and there is at least one element of A that is not an element of B. That is for every set A, $A \subseteq A$ is true, but $A \subset A$ is false.

3. Union and intersection

Specify the following sets. (Probably listing the members will be easiest, but other ways are of course welcome too.)

(a) $A \cap C$
(b) $B \cap C$
(c) $B \cap E$
(d) $F \cup A$
(e) $F \cup C$
(f) $F \cap A$
(g) $D \cap B$
(h) $D \cup B$

4. The empty set

Are the following statements true, false, or ill-formed?

(a) $E \subseteq C$
(b) $E \in C$

5. Difference

Specify the following sets.

(a) $A - F$
(b) $F - A$
(c) $D - B$
(d) $B - D$

6. Complement

Assume the set of all people in this class is U, your "discourse universe". What's the following set?

(a) $C(\{ x \mid x$'s name has more than two letters $\})$

7. Power Set

Specify the following sets.

(a) $\wp(\{$ Ginger, Greg, Gillian $\})$
(b) $\wp(\{ x \mid x$ is a natural number greater than 3 and smaller than 6 $\})$

8. Complement vs. difference

Sometimes the difference operation '–' is called the 'relative complement'. Can you explain this? Try to incorporate the requirement that we talk about members of U into the definition of complement and compare it to the definition of difference.