1. Different ways of specifying sets

Give a different notation that picks out the same set.

(a) { 2, 4, 6, 8 }
{ x | x < 10 and $^{x}{}_{2}$ is a natural number }
{ x | x is an even number between 1 and 9 }
…

(b) { x | x^2 = 9 }
{ 3, -3 }
{ x | x = 3 or x = -3 }

(c) { Ginger, Greg, Gillian }
{ x | x is taking Ling510 this semester and x's name starts with a 'G' }

For the following exercises, use these sets:

A = { Q, Patrick, Elisabeth, Jack }
B = { 2, 4, 6, 8 }
C = { {Patrick}, 2, {the Berlin Wall, the Hampshire Mall} }
D = { Q, 2, 4, Jack, {Elisabeth} }
E = $\emptyset$
F = { Patrick }

2. Members and subsets

Are the following statements true, false, or ill-formed?

(a) Patrick $\in$ F
true

(b) Patrick $\in$ C
false. { Patrick } $\in$ C but not Patrick

(c) {Patrick} $\in$ F
false

(d) {Patrick} $\in$ C
true

(e) D $\subseteq$ A
false

(f) B $\subseteq$ D
false

(g) F $\subseteq$ C
false. { Patrick } $\in$ C but not { Patrick } $\subseteq$ C, because Patrick $\notin$ C

(h) F $\subseteq$ A
true

(i) the Berlin Wall $\subseteq$ C
ill-formed. the definition of $\subseteq$ states that "A $\subseteq$ B iff every member of A …" – since the Berlin Wall is not a set, we don't know what to make of "every member of the Berlin Wall …"

(j) {the Berlin Wall} $\subseteq$ C
false

(k) the Berlin Wall $\in$ C
false

(l) {the Berlin Wall} $\in$ C
false

3. Union and intersection

Specify the following sets. (Probably listing the members will be easiest, but other ways are of course welcome too.)

(a) A $\cap$ C
$\emptyset$ (note: {Patrick} $\neq$ Patrick)

(b) B $\cap$ C
{2}

(c) B $\cap$ E
$\emptyset$

(d) F $\cup$ A
{Q, Patrick, Elisabeth, Jack}

(e) F $\cup$ C
{Patrick, {Patrick}, 2, {the BW, the HM}}

(f) F $\cap$ A
{Patrick}

(g) D $\cap$ B
{2, 4}

(h) D $\cup$ B
{2, 4, 6, 8, Q, Jack, {Elisabeth}}

4. The empty set

Are the following statements true, false, or ill-formed?

(a) E $\subseteq$ C
this comes out true, because of the way "every" is used in logic and math: every x is P is true if there are no x. so every member of E is a member of C is true since there aren't any members in E.

(b) E $\in$ C
false

5. Difference

Specify the following sets.

(a) A – F
{Q, Jack, Elisabeth}

(b) F – A
$\emptyset$

(c) D – B
{Q, Jack, {Elisabeth}}

(d) B – D
{6, 8}

6. Complement

Assume the set of all people in this class is U, your "discourse universe". What's the following set?

(a) *C*({ x | x's name has more than two letters })
{Q}

7. Power Set

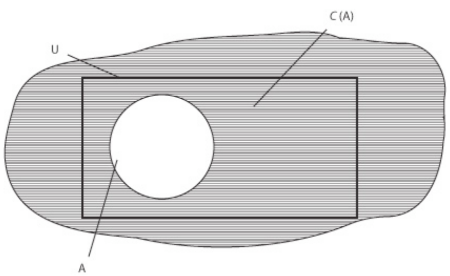
Specify the following sets.

(a) $\wp$({ Ginger, Greg, Gillian })
{$\emptyset$,{Gr.},{Gin.},{Gil.},{Gr.,Gin.},{Gr.,Gil.},{Gin.,Gil.},{Gr.,Gin.Gil.}}

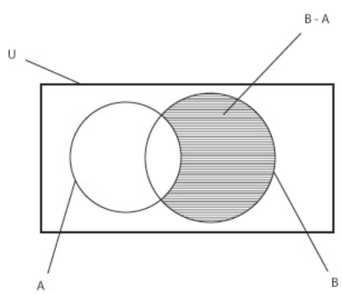
(b) $\wp$({ x | x is a natural number greater than 3 and smaller than 6 })
{$\emptyset$,{4},{5},{4,5}}

8. Complement vs. difference

Sometimes the difference operation '–' is called the 'relative complement'. Can you explain this? Try to incorporate the requirement that we talk about members of U into the definition of complement and compare it to the definition of difference.



When we talk about the complement of A, written as *C*(A), A⁻, or A', we talk about everything that is not in A, that is {x | x ∉ A}. We limit out attention to a set of objects that we presume or hypothesize are the only ones to exists for our current purposes, that's out universe 'U'. Hence the complement is inherently limited to talk about things in U, and we could write it equally well as U – A , the set of things in U but not in A.



In the case of the difference between say B and A, B – A, we can picture B – A as the complement of A relative to the union of A and B, A ∪ B.
(A ∪ B) – A is equivalent to B – A.