

## 0. Things to keep in mind

$\langle a,b \rangle$  is an ordered pair with  $a$  as its first and  $b$  as its second member. Its distinct from  $\langle b,a \rangle$ .

$A \times B =_{\text{def}} \{ \langle x,y \rangle \mid x \in A \text{ and } y \in B \}$ .  $A \times B$  is called the Cartesian Product of  $A$  and  $B$ .

$R$  is a relation from  $A$  to  $B$  iff  $R \subseteq A \times B$ .

$\text{domain}(R) =_{\text{def}} \{ x \mid \text{for some } y, \langle x,y \rangle \in R \}$

$\text{range}(R) =_{\text{def}} \{ y \mid \text{for some } x, \langle x,y \rangle \in R \}$

Some relations from  $A$  to  $B$  will be functions from  $A$  to  $B$ , namely those where each element of  $A$  is paired with exactly one element from  $B$ .

Example of a relation:

$\{ \langle \text{Anne}, \text{Mo} \rangle, \langle \text{Bill}, \text{Franz} \rangle, \langle \text{Anne}, \text{Franz} \rangle, \langle \text{George}, \text{Mo} \rangle \}$

(This might be the relation  $x$  is a brother of  $y$  between Anne, Mo, Bill, George, and Franz.)

Example of a function:

$\{ \langle 1,3 \rangle, \langle 2,4 \rangle, \langle 3,5 \rangle, \langle 4,6 \rangle, \langle 5,7 \rangle \}$

(This might be the function  $y = x+2$  limited to the domain  $1, 2, 3, 4, 5$ .)

## 1. Cartesian Products and Relations

Assume the sets  $A = \{1,2\}$  and  $B = \{a,b,c\}$ .

What are

- (a)  $A \times B$                       (b)  $B \times A$                       (c)  $A \times A$

Assume the relation  $R = \{ \langle a,1 \rangle, \langle a,2 \rangle, \langle c,1 \rangle \}$ .

What is  $R$  a relation from and to?

Give an example of a relation in  $A$ .

## 2. Cars and people

Assume the sets  $C = \{ \text{Ford Escort}, \text{Jeep}, \text{Minivan} \}$  and  $P = \{ \text{Lucy}, \text{Dave}, \text{Briana} \}$ .

- (a) Give an example of a relation from  $P$  to  $C$ .  
(b) Is the example you gave a relation or a function? Why?  
(c) What are some examples of "natural" relations and functions that you can imagine between  $C$  and  $P$ .

## 3. Phone numbers

What kinds of things would be in the domain of the relation  $\{ \langle x,y \rangle \mid x \text{ is the phone number of } y \}$ .

What kinds of things in the range? Would it be a function? Why or why not?