There are many more answers you could have given besides the ones listed here. Please let me know if you have any questions.

1. Cartesian Products and Relations

Assume the sets A = {1,2} and B = {a,b,c}.
What are

(a) A × B
{⟨1,a⟩, ⟨1,b⟩, ⟨1,c⟩, ⟨2,a⟩, ⟨2,b⟩, ⟨2,c⟩}

(b) B × A
{⟨a,1⟩, ⟨a,2⟩, ⟨b,1⟩, ⟨b,2⟩, ⟨c,1⟩, ⟨c,2⟩}

(c) A × A
{⟨1,1⟩, ⟨1,2⟩, ⟨2,1⟩, ⟨2,2⟩}

Remember, the first elements of the pairs ⟨a,b⟩ in R are members of the first set (from, domain), while the second elements are from the second set (to, range).

Assume the relation R = { ⟨a,1⟩, ⟨a,2⟩, ⟨c,1⟩ }.
What is R a relation from and to? R is a relation from A to B.

Give an example of a relation in A. f.i. {⟨1,1⟩, ⟨2,2⟩} (this would be the relation “is identical to”)
{⟨1,2⟩, ⟨2,1⟩} (this would be “is not identical to”)

2. Cars and people

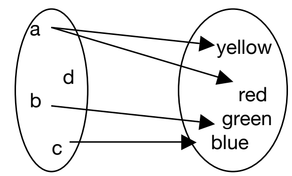
Assume the sets C = { Ford Escort, Jeep, Minivan } and P = { Lucy, Dave, Briana }.

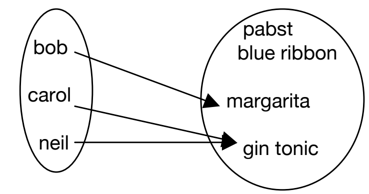
(a) Give an example of a relation from P to C.
{⟨Lucy,Jeep⟩, ⟨Lucy,Minivan⟩, ⟨Lucy,Ford Escort⟩, ⟨Dave,Jeep⟩}

(b) Is the example you gave a relation or a function? Why?
The example above is a relation. There are two facts, each of them shows that it is not a function:
- There is an element in the domain that appears paired with several elements of the range: Lucy (in ⟨Lucy,Jeep⟩, ⟨Lucy,Minivan⟩, and ⟨Lucy,Ford Escort⟩), and
- there is an element in the domain that is not paired with any element of the range: Briana.[1]

A function is a particular kind of relation, namely one in which every element of the domain is paired up with exactly one element from the range.

You can think of possible relations from A to B as all the ways in which you can draw arrows from elements of A pointing to elements of B. The two diagrams below illustrate two particular relations.





Of those two relations, only the one on the right is a function. Functions have exactly one outgoing arrow for every element on the left, while relations have no such restrictions. There might be elements on the left with multiple outgoing arrows, or ones with none.

(c) What are some examples of “natural” relations and functions that you can imagine between C and P.
Some relations:
{⟨x,y⟩ | x is the driver of y} (a relation from P to C)
{⟨x,y⟩ | x is owned by y} (a relation from C to P)
For functions we have to impose some restrictions:
In a situation where everybody admires one and only one car
{⟨x,y⟩ | x is admired by y} would be a function from C to P, and
in a situation where everybody in P is currently driving, and nobody can drive more than one car
{⟨x,y⟩ | x is currently driving y}

3. Phone numbers

What kinds of things would be in the domain of the relation { ⟨x,y⟩ | x is the phone number of y }. What kinds of things in the range? Would it be a function? Why or why not?

{ ⟨x,y⟩ | x is the phone number of y } is a relation from phone numbers to phone company customers (could be individuals, or businesses, let’s not fuss about that).
It would be a function, assuming that exactly one phone customer is responsible for every phone number given out (which seems reasonable – no idle numbers, and if people share a phone line, only one person would actually sign up).

[1] Keeping in mind Brian’s remark that we only consider total functions so far.